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# MESH MODEL 1964

## OFFICE NOTE 24

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The so-called "mesh model" used at NMC until 1962 is thoroughly described in JNWP Office Note No. 15, 1960, which will hereafter be referred to as "O.N. 15." The 500 mb barotropic forecast in this model was made in exactly the same manner as in the single level 500 mb barotropic model. There was no feed-back from other levels at any stage. Cressman's [1] mountain and surface friction terms were computed using the 500 mb stream function winds reduced by a factor depending only on the height of the terrain. The type of errors which might have arisen from this crude estimate of the surface winds was discussed by Fawcett [2].

The primary reason for reprogramming the mesh model was to use the 850 mb-500 mb thickness forecasts to obtain a better estimate of the surface winds. This, of course, required that the two forecast equations be integrated as a marching jury problem, each requiring output from the other at each time step. Impetus for the improvement and revival of the mesh came in July 1964 when the RADAT radiosonde report was expanded to include 850 mb as well as 500 mb data. This made it possible to use the mesh model for the preliminary 1+30 numerical forecast which had previously been made with the 500 mb-only barotropic model. The difference in running time between the mesh and the single level barotropic is only three minutes for a 36-hour forecast on the IBM 7094-II.

Except for the divergence term due to terrain effects in the 500 mb vorticity equation, the new mesh model is identical in principle with the old. The thickness tendency equation (Equation 3.16 in O.N. 15), for example, was not changed and the 500 mb forecast remains basically barotropic in nature. Several refinements in the computational program were made. The only one which is significant in yielding improved results, however, is the incorporation of truncation error control in the Jacobian computations. This control which is described in NMC Bulletin No.35* had previously been used in other NMC operational models.

In the previous mesh model the surface wind $\mathbb{W}_s$ was estimated as:

$$\mathbb{W}_s = \mathbb{W}_5 \left[ 1 - 0.8 \left( \frac{p_s - 500}{500} \right) \right]$$

where $\mathbb{W}_5 = K \times \nabla \psi_5$ is the 500 mb wind vector and $p_s$ is the standard

*NMC Bulletin No.35 is attached as Appendix to this Note.

surface pressure.

In the new model this is replaced by:

(1) $\mathbb{V}_s = \mathbb{V}_5 - \left(\frac{p_s - 500}{350}\right) S_g$

where $S_g = g/f\ K \times \nabla h$ is the 850-500 mb geostrophic shear

and $h$ = 850-500 mb thickness.

Letting $R = \frac{g}{f}\left(\frac{p_s - 500}{350}\right)$ equation (1) can be written:

(2) $\mathbb{V}_s = K \times (\nabla\psi_5 - R\nabla h)$

The vertical motion due to the mountains $\omega_M$ now becomes:

(3) $\omega_M = \mathbb{V}_s \cdot \nabla p_s = J(\psi_5, p_s) - RJ(h, p_s)$

The vertical motion due to surface friction, $\omega_H$, is given in Cressman's [1] equation (10) as:

(4) $\omega_H = \frac{\rho g}{f}\left[\frac{\partial}{\partial y}(C_d V_s u_s) - \frac{\partial}{\partial x}(C_d V_s v_s)\right]$

where $V_s^2 = u_s^2 + v_s^2 = \mathbb{V}_s \cdot \mathbb{V}_s$

Equation (4) can also be written as:

(5) $\omega_H = \frac{\rho g V_s}{f}\left[-C_d \zeta_s + V_s \frac{\partial C_d}{\partial n} + C_d \frac{\partial V_s}{\partial n}\right]$

where $\zeta_s = \frac{\partial v_s}{\partial x} - \frac{\partial u_s}{\partial y} = KV_s - \frac{\partial V_s}{\partial n}$ is the surface vorticity.

The second term inside the brackets of (5) represents the effect due to the variation in the drag coefficient, $C_d$, normal to the surface flow. Although this term is no doubt locally significant, its effects are of a smaller scale than those due to the surface vorticity term. It is also quite doubtful whether this effect can be properly simulated in a model such as this. The third term in the brackets duplicates the shear part of the vorticity term and reinforces it. It arises from assuming the surface stress proportional to the <u>square</u> of the wind speed rather than more common linear relation. Forecasts of the surface shear are also less accurate than of the vorticity which is more conservative. In view of these considerations only the first term inside the brackets of (5) is retained. Using equation (2) to compute $\zeta_s$, we obtain:

$$\text{(7)} \qquad \omega_H = \frac{1}{f} \rho g C_d V_s (\nabla^2 \psi_5 - R \nabla^2 h)$$

In deriving (7) the slight variation in R due to latitude has been neglected. It is essential that the variation due to $p_s$ be neglected, otherwise, the vertical shear would be erroneously included in $\zeta_s$.

As in the previous mesh and barotropic models, it is assumed that the divergence at 500 mb $(DIV)_5$, is given by:

$$(DIV)_5 = - \frac{\omega_M + \omega_H}{p_s - 200 \text{ mb}}$$

In this new model, however, the divergence term is linearized by replacing $\eta_5 = f + \zeta_5$ with $f = 2\Omega \sin \Phi$.

It is not possible to say which of these options is more theoretically correct. When integrated over an area, the sum of the vorticity advection and the terrain divergence terms does not yield a line integral along the boundary in either case. Thus, an analysis of systematic errors such as that of Wiin-Nielson [3] does not apply. The choice of (f) rather than ($\eta$) for this new model was based on experience which showed that the use of $\eta$ could lead to spurious deepening of a stationary low.

The vorticity equation for 500 mb with this change then becomes:

$$\text{(8)} \qquad \left(\nabla^2 - \frac{\mu \eta_5}{\bar{\psi}}\right) \frac{\partial \psi_5}{\partial t} = - J(\psi_5, \eta_5) + f \left(\frac{\omega_M + \omega_H}{p_s - 200}\right)$$

In finite-difference form and with scaled variables equation (8) transforms to:

$$\text{(9)} \qquad \left(\nabla^2 - .6093 \frac{\hat{\eta}}{\hat{m}^2}\right) \Delta_T \hat{\psi} = - 4 J(\hat{\psi}, \hat{\eta}) + \frac{4\hat{f}(M' + F')}{(\hat{p} - .1)}$$

where: $\hat{\eta} = 1.5444 \, \hat{m}^2 \nabla^2 \hat{\psi} + \hat{f}$

$\hat{f} = .065629 \sin \Phi$

$\hat{m} = \frac{1}{2} m = \frac{1}{2} \left(\frac{1 + \sin 60°}{1 + \sin \Phi}\right)$

$\hat{\psi} = \frac{\hat{f}}{g} 2^{-17} \psi_5$ ($\psi_5$ in centimeters)

$$\overline{f} = 2\Omega \sin 45 = 1.03125 \times 10^{-4} \text{ sec}^{-1}$$

$$\hat{p} = \tfrac{1}{2} p_s \times 10^{-3} \qquad (p_s \text{ in millibar})$$

$$\Delta\tau = 7200 \frac{\partial}{\partial t}$$

$$\nabla^2 = \frac{d^2}{m^2} \nabla^2$$

$$J = \frac{4d^2}{m^2} J$$

$$M' = J(\hat{\psi}, \hat{p}) - 4r' J(\hat{h}, \hat{p})$$

$$F' = \frac{-66\,m}{\sin \emptyset} Q \left\{ C_d \sqrt{(\Delta_x \hat{\psi} - 4r'\Delta_x h)^2 + (\Delta_y \hat{\psi} - 4r'\Delta_y h)^2} \right\}$$

$$r' = \frac{.3536}{\sin \emptyset} \left( \frac{\hat{p} - .25}{.35} \right)$$

$C_d$ = drag coefficient as in Ref. [1]

$$Q = 4(\nabla^2 \hat{\psi} - 4r' \nabla^2 \hat{h})$$

$$\Delta_x \hat{\psi}_{ij} = \hat{\psi}_{i+1,j} - \hat{\psi}_{i-1,j}$$

Equation (9) should be compared with the final equation in O. N. 15 (Page 36) which was:

$$(10) \quad \left( \nabla^2 - .942 \frac{\hat{\eta}}{\hat{M}^2} \right) \Delta_\tau \hat{\psi} = -4.52 J(\hat{\psi}, \hat{\eta}) + 4 \frac{\hat{\hat{\eta}}}{(\hat{p} - .1)} (M_n + F)$$

where $\hat{M}^2 = \dfrac{m^2}{2.5898}$

$$M_n = r J(\hat{\psi}, \hat{p})$$

$$F = \frac{-66\,m\,r^2}{\sin \emptyset} \left[ \delta_x (C_d \hat{V}_5 \Delta_x \hat{\psi}) + \delta_y (C_d \hat{V}_5 \Delta_y \hat{\psi}) \right]$$

$$r = 1 - .8 \left( \frac{\hat{p} - .25}{.25} \right)$$

$$\hat{V}_5 = \sqrt{(\Delta_x \hat{\psi})^2 + (\Delta_y \hat{\psi})^2}$$

$$\delta_x (\ )_{i,j} = (\ )_{i+1,j} - (\ )_{i-1,j}$$

The following differences between equations (9) and (10) will be noted:

(a) The coefficient in the Helmholtz term has been changed corresponding to the change in the scaling of the map factor squared from $\hat{M}^2$ to $\hat{m}^2$. In the previous mesh, different scalings were used for $m^2$; $\hat{M}^2$ was used in the vorticity equation and $\hat{m}^2$ was used in the thickness tendency equation.

(b) The coefficient in the vorticity advection term, which was $4.52 = 4 \times 1.13$ has been changed to 4 corresponding to the use of truncation error control in calculating the Jacobian.

(c) The mountain and friction terms have been changed as discussed above. The factor 4 preceding the parenthesis in the definition of $Q$ enters since the Laplacians here are single mesh whereas previously the corresponding second derivatives were computed over a double mesh length.

(d) As previously mentioned the divergence term has been linearized.

The thickness tendency equation was not changed and its finite difference form is identical with that described in O. N. 15, namely:

$$(11) \quad \overset{*}{\hat{\eta}} = 2A\hat{m}^2 \nabla^2 \hat{\psi} + B \sin \phi$$

$$(12) \quad \left(\nabla^2 - 2C \frac{\overset{*}{\hat{\eta}} \sin \phi}{\hat{m}^2 \hat{\sigma}^2}\right) \overset{*}{\nabla}_T h = -4B \sin \phi \, J\left(\hat{\psi}, \frac{\hat{m}^2}{\sin \phi} \nabla^2 \hat{h}\right)$$

$$- 2J(\hat{h}, \overset{*}{\hat{\eta}}) + 4 \sin \phi \frac{\overset{*}{\hat{\eta}}}{\hat{\sigma}^2} J(\hat{\psi}, \hat{h}).$$

where
$A = .5792$
$B = .065629$
$C = .8633$
$\hat{h} = h \times 2^{-17}$
$\overset{*}{\Delta}_T = 3600 \frac{\partial}{\partial t}$
$\hat{\sigma}^2 = 2^{-28}(1.9702)\sigma^2 = \begin{cases} .411 \text{ in winter} \\ .316 \text{ in summer} \\ .363 \text{ in spring and fall} \end{cases}$

It should be noted that $\overset{*}{\eta}$ refers to a mean level where $\overline{\psi}$ is assumed to be $\frac{3}{4}(\psi_5)$ and $\overline{\eta} = \frac{3}{4}\zeta_5 + f$.

The factor $2C$ in the Helmholtz term of equation (12) appeared as $C$ in equation 3.28 of O. N. 15 (Page 26). This was evidently a typographical error.

Although different $\Delta_T$'s are used in equations (9) and (12), the time steps for both are 2 hour "centered difference" except for the initial one hour "uncentered" step. Both $\hat{\psi}$ and $\hat{h}$ are smoothed at 12 hour intervals using Shuman's [4] 9 point operator.

The model was programmed for the IBM 7094 by Messrs. M. P. Snidero and R. Schnurr. It was put into operation December 1, 1964, replacing the 500 mb only barotropic model in the 1+30, "RADAT" forecast.

# REFERENCES


1. Cressman, G. P., "Improved Terrain Effects in Barotropic Forecasts," Monthly Weather Review, Vol. 88, Nos. 9-12, September-December 1960.

2. Fawcett, E. B., "A Study of Errors in Barotropic Forecasts of Cut-Off Lows in the Southwestern United States During Fall and Winter of 1961-1962," Technical Note No. 8, Office of Forecast Development, USWB, May 1962.

3. Wiin-Nielson, A., "On Certain Integral Constraints for the Time-Integration of Baroclinic Models," Tellus, Vol. 11, No. 1, February 1959.

4. Shuman, F. G., "Numerical Methods in Weather Prediction: II: Smoothing and Filtering," Monthly Weather Review, Vol. 85, November 1957.